Putting Bayes to sleep

Wouter M. Koolen Dimitri Adamskiy Manfred Warmuth

Royal Holloway University of London

University of California Santa Cruz

WITMSE

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Menu

How
a beautiful and intriguing piece of technology
provides
new insights in existing methods
and
improves the state of the art
in
expert tracking
and
online multitask learning

Point of departure

We are trying to predict a sequence $y_1, y_2, \ldots$.

Definition

A **model** issues a prediction each round. We denote the prediction of model m in round t by $P(y_t|y_{<t}, m)$.

Say we have several models $m = 1, \ldots, M$.
How to combine their predictions?

Bayesian answer

Place a **prior** $P(m)$ on models. Bayesian **predictive distribution**

$$P(y_t|y_{<t}) \;=\; \sum_{m=1}^{M} P(y_t|y_{<t}, m)P(m|y_{<t})$$

where **posterior distribution** is incrementally updated by

$$P(m|y_{\leq t}) \;=\; \frac{P(y_t|y_{<t}, m)P(m|y_{<t})}{P(y_t|y_{<t})}$$

Bayes is **fast**: predict in $\mathcal{O}(M)$ time per round.

Bayes is **good**: regret w.r.t. model m on data $y_{\leq T}$ bounded by

$$\sum_{t=1}^{T} \left(-\ln P(y_t|y_{<t}) + \ln P(y_t|y_{<t}, m)\right) \;\leq\; -\ln P(m).$$

Specialists

Definition (FSSW97,CV09)

A **specialist** may or **may not** issue a prediction.
Prediction $P(y_t|y_{<t}, m)$ only available for **awake** $m \in W_t$.

Say we have several specialists $m = 1, \ldots, M$.
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If we **imagine** a prediction for the sleeping specialists, we can use Bayes

Choice: sleeping specialists predict with Bayesian predictive distribution

That **sounds** circular. Because it **is**. \$%!#?

Bayes for specialists

Place a **prior** $P(m)$ on models. Bayesian **predictive distribution**

$$P(y_t|y_{<t}) \;=\; \sum_{m \in W_t} P(y_t|y_{<t}, m) P(m|y_{<t}) + \sum_{m \notin W_t} P(y_t|y_{<t}) P(m|y_{<t})$$

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The **posterior distribution** is incrementally updated by

$$P(m|y_{\leq t}) \;=\; \begin{cases} \frac{P(y_t|y_{<t}, m)P(m|y_{<t})}{P(y_t|y_{<t})} & \text{if } m \in W_t, \\ \frac{P(y_t|y_{<t})P(m|y_{<t})}{P(y_t|y_{<t})} = P(m|y_{<t}) & \text{if } m \notin W_t. \end{cases}$$

Bayes is **fast**: predict in $\mathcal{O}(M)$ time per round.

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The specialists trick

Specialists just a curiosity?

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Specialists trick:

- Start with input **models**
- Create many derived **virtual specialists**
  - Control how they predict
  - Control when they are awake
- Run Bayes on all these specialists
- Carefully choose prior
  - Low regret w.r.t. class of intricate combinations of input models.
  - Fast execution. Bayes on specialists **collapses**

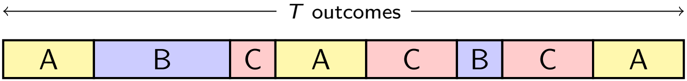
Application 1: Freund's Problem

In practice:

- Large number M of models
- Some are good some of the time
- Most are bad all the time

We do not know in advance **which** models will be useful **when**.

Goal: compare favourably on data $y_{\leq T}$ with the best alternation of $N \ll M$ models with $B \ll T$ blocks.

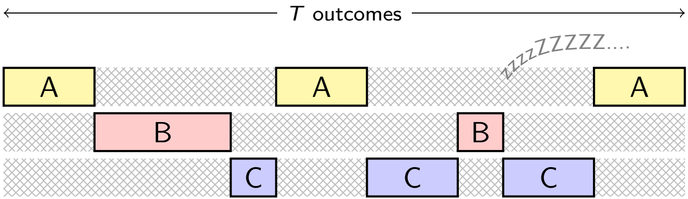


partition model, $N = 3$ good models, $B = 8$ blocks

Bayesian reflex: average all partition models. **NP hard**.

Application 1: sleeping solution

Create partition specialists



Bayes is **fast**: $\mathcal{O}(M)$ time per trial, $\mathcal{O}(M)$ space
Bayes is **good**: regret close to information-theoretic lower bound

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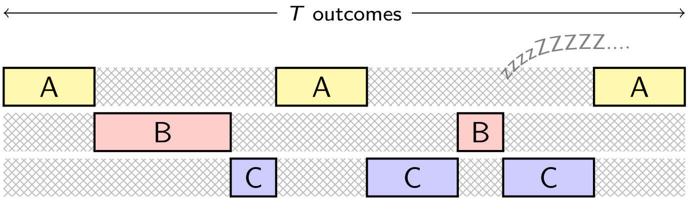

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- Bayesian interpretation for Mixing Past Posteriors
- Faster algorithm
- Slightly improved regret bound
- Explain mysterious factor 2

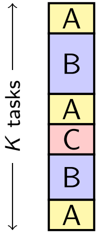
Application 2: online multitask learning

In practice:

- Large number M of models
- Data $y_1, y_2, \ldots$ from K interleaved tasks
- Observe task label $\kappa_1, \kappa_2, \ldots$ before prediction

We do not know in advance **which** tasks are **similar**.

Goal: Compare favourably to best clustering of task with $N \ll K$ cells

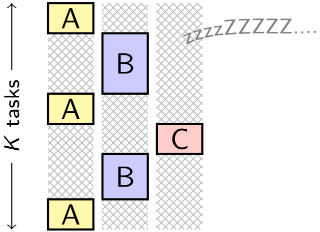


cluster model, $N = 3$ cells

Bayesian reflex: average all cluster models. **NP hard**.

Application 2: sleeping solution

Create cluster specialists



Bayes is **fast**: $\mathcal{O}(M)$ time per trial, $\mathcal{O}(MK)$ space
Bayes is **good**: regret close to information-theoretic lower bound

- Intriguing algorithm
- Regret independent of task switch count

Conclusion

The **specialists trick**

- NP-hard offline problem
- Ditch Bayes on **coordinated** comparator models
- Instead create **uncoordinated** virtual specialists
- Craft prior so that
  - Bayes collapses (efficient emulation)
  - Small regret

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Thank you!