Our new algorithm Squint

- adapts to the difficulty of the learning problem by learning the learning rate,
- thereby integrating both the popular second-order and quantile adaptivities,
- at the run time of standard Hedge.

\[ R_k^T < \sqrt{V_k^T \ln K} \] for each expert \( k \)

for some second-order \( V_k^T \leq L_k^T \leq T \)

- stochastic case, learning sub-algorithms
- specialized algorithms, hard-coded in K.

Three priors

1. Uniform prior (generalizes to conjugate)
\[ \gamma(\eta) = 2 \]
Efficient algorithm, \( C_T = \ln V_f^T \).
2. Chernov-Vovk (2010) prior
\[ \gamma(\eta) = \frac{\ln 2}{\eta \ln^2(\eta)} \]
Not efficient, \( C_T = \ln V_f^T \).
3. Improper(!) log-uniform prior
\[ \gamma(\eta) = \frac{1}{\eta} \]
Efficient algorithm, \( C_T = \ln V_f^T \).

Extensions

Combinatorial concept class \( C \subseteq \{0,1\}^K \):
- Shortest path
- Spanning trees
- Permutations

Component Squint guarantees:
\[ R_k^T < \sqrt{V_f^T (\text{comp}(u) + K C_T)} \] for each \( u \in \text{conv}(C) \).

The reference set of experts \( K \) is subsumed by an “average concept” vector \( u \in \text{conv}(C) \), for which our bound relates the coordinate-wise average regret \( R_u^T = \sum_k u_k r_k^T \) to the averaged variance \( V_f^T = \sum_k u_k (r_k^T)^2 \) and the prior entropy \( \text{comp}(u) \).

Future work

Loss range adaptivity, bandits, online convex optimization

Hedge setting

\( K \) experts

In round \( t = 1, 2, \ldots \)
- Learner plays a probability distribution \( w_t = (w_{t1}, \ldots, w_{tk}) \) on experts
- Adversary reveals the expert loss vector \( \epsilon_t = (\epsilon_{t1}, \ldots, \epsilon_{tk}) \in [0,1]^k \)
- Learner incurs loss \( w_t^T \epsilon_t \)

The goal is to have small regret
\[ R_k^T := \sum_{t=1}^T w_t^T \epsilon_t \]

with respect to every expert \( k \) at every time \( T \).

Second-order adaptivity

Choose \( k \) and fix \( \gamma = \frac{R_k^T}{2V_k^T} \). Now as
\[ 1 \geq \Phi_T \geq \pi(k) \gamma(\eta) e^{R_k^T - \eta V_k^T} = \pi(k) \gamma(\eta) e^{\frac{R_k^T}{2V_k^T}} \]
we have
\[ R_k^T \leq 2 \sqrt{V_f^T (\ln \pi(k) - \ln \gamma(\eta))} \]
For quantile bound take \( \sum_{k \in K} \)

Quantile adaptivity

Prior \( \pi \) on experts:
\[ \min_{k \in K} R_k^T < \sqrt{T \ln \pi(K)} \] for each subset \( K \) of experts

- over-discretization, company baseline
- specialized algorithms, hard-coded

“Impossible tunings”. Efficiency.

Squint guarantees both

Squint algorithm with bound
\[ R_k^T < \sqrt{V_k^T (-\ln \pi(K) + C_T)} \] for each subset \( K \) of experts

where \( R_k^T = E_{\pi(K)} R_k^T \) and \( V_k^T = E_{\pi(K)} V_k^T \)
denote the average (under the prior \( \pi \)) among the reference experts \( k \in K \) of the cumulative regret \( R_k^T = \sum_{t=1}^T r_t^k \) and the (uncentered) variance of the excess losses \( V_k^T = \sum_{t=1}^T (r_t^k)^2 \) (where \( r_t^k = (w_t - e_t^k)^T \epsilon_t^k \).

Pretty two-line proof

Squint potential motivation

Fix prior \( \pi \) on experts \( k \in \{1, \ldots, K\} \) and prior \( \gamma \) on learning rates \( \eta \in [0,1/2] \).

Potential function (weighted sum of objectives)
\[ \Phi_k := \mathbb{E}_{\pi(k)}[\gamma (e^{R_k^T - \eta V_k^T})] \]

and associated weights
\[ w_{t+1} := \pi(k) E_{\pi(k)}[\gamma (e^{R_k^T - \eta V_k^T})] \]

constant time per expert per round

Regret guarantee

Choose \( k \) and fix \( \gamma = \frac{R_k^T}{2V_k^T} \). Now as
\[ 1 \geq \Phi_T \geq \pi(k) \gamma(\eta) e^{R_k^T - \eta V_k^T} = \pi(k) \gamma(\eta) e^{\frac{R_k^T}{2V_k^T}} \]
we have
\[ R_k^T \leq 2 \sqrt{V_f^T (\ln \pi(k) - \ln \gamma(\eta))} \]
For quantile bound take \( \sum_{k \in K} \)

Classic Hedge result

The Hedge algorithm with learning rate \( \eta \)
\[ w_{t+1} := \frac{e^{-|\epsilon|_1}}{\sum e^{-|\epsilon|_1}} \quad \text{where} \quad L_k^T = \sum_{t=1}^T e_t^k \]

upon proper tuning of \( \eta \) ensures
\[ R_k^T < \sqrt{T \ln K} \] for each expert \( k \).

Tight for adversarial (worst-case) losses. Underwhelming in practice.